Hölder regularity for collapses of point vortices

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We study the collapses of point-vortices for the Euler equation in the plane and for surface quasi-geostrophic equations in the general setting of α models. In these models, the kernel of the Biot-Savart law is a power function of exponent $-\alpha$. It is proved that, under a standard non-degeneracy hypothesis, the trajectories of the point-vortices have a Hölder regularity up to the time of collapse. The Hölder exponent obtained is $1/(\alpha+1)$ and this exponent is proved to be optimal for all α by exhibiting an example of a 3-vortex collapse.

The same question is then addressed for the Euler point-vortex system in smooth bounded connected domains. It is proved that if a given point-vortex has an accumulation point in the interior of the domain near the time of collapse, then it converges towards this point and displays the same Hölder continuity property. A partial result for point-vortices that collapse with the boundary is also established : we prove that their distance to the boundary is Hölder regular.

Joint work with Martin DONATI.